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Comparative of Interpolators Applied to Depth Images

Beatriz Juárez Arreortúa, Hugo Jiménez Hernández, and
Diana Margarita Córdova Esparza

Center for Engineering and Industrial Development, Querétaro, Qro., México
{beatriz.juareza, hugojh, diana_mce}@gmail.com

Abstract. Interpolators are widely used in image processing because they allow us to estimate the unknown values of sensor measurements. In this research, we present a comparison between commonly used interpolators to evaluate how each affect the behavior of the data, it is studied the problem of interpolation as a means to infer information at a higher frequency through the mathematical description an depth image.

Keywords: Interpolation, Depth images, Frequency.

1 Introduction

Image processing and analysis is currently better known and used for various activities in the world of technology, the type of analysis that is performed and the techniques used are directly linked to the amount of information provided by each image, as well as what needs to be identified through this one.

This information depends in turn on the sampling frequency of the sensor being used; a digital image is constituted by a spatial sampling of a set of sensors represented by a matrix. However, when talking about sampling, the main limitation observed is the sensor acquisition frequency, which in turn is limited due to the characteristics of the phenomenon being sampled. In several occasions to compensate for the limitations above, it is common the use of interpolators, so that we can obtain more information about the phenomenon being analyzed.

Therefore, when using an interpolator it is expected that the information obtained through it will be consistent with the data originally acquired, that is, that it does not deform the nature of the information, since this can directly affect the result of the analysis performed.

Therefore, in this work, we study the problem of interpolation as a means to infer information at a higher frequency. We examined how three different types of interpolators affect the data acquired at a certain frequency. We present experiments with depth images of increased resolution and analyze how much it affects the method of interpolation used in the original image.

2 Theoretical Foundation

There is evidence in the literature of different comparatives between interpolators, however these are based on criteria such as: execution time, precision, clarity of the image, among others [1], [2]. However, for purposes of this work, what is interesting is to know how much it affects or not the use of some interpolator to the original distribution of the depth image.

In this work, three commonly used interpolators are used in image analysis, which are described below.

2.1 Linear Interpolation

One of the most used interpolators is the linear one described in Eq. (1) due to its simplicity. It consists in fitting a line to two given points:

$$g(x) = \frac{b-x}{b-a} f(a) + \frac{x-a}{b-a} f(b). \quad (1)$$

where g (x) denotes that this is a first-degree interpolation polynomial. To interpolate an image, the function is first applied to the x-axis and then to the y-axis.

The advantage of using the linear interpolator is that the implementation is simple, for this reason, the computation time is small compared to other interpolators. Another advantage of the linear interpolation is that the results are more accurate with smaller intervals between the two points. However, in the same way, if the interval is large, the result is more inaccurate. It should also be considered that if the selected points do not correspond to a straight line, the calculated values become incorrect.

2.2 Lagrange Interpolation

The Lagrange interpolation polynomial is a reformulation of Newton's polynomial that avoids the calculation of the divided differences, and is represented by Eq. (2) of polynomial bases of Lagrange (Eq. 3):

$$f(x) = \sum_{j=0}^{k} y_i l_i(x). \quad (2)$$

$$l_j(x) = \prod_{i=0,\ i\neq j}^{n} \frac{x-x_i}{x_j-x_i} = \frac{x-x_0}{x_j-x_0} \cdots \frac{x-x_0}{x_j-x_0} \frac{x-x_0}{x_j-x_0} \cdots \frac{x-x_0}{x_j-x_0}. \quad (3)$$

The Lagrange interpolation grows fast computationally with the increase of the interpolator degree. The polynomial degree varies according to the input points, i.e., if we remove or add points it is necessary to change the degree of the polynomial.

2.3 Basic Splines (B-Splines)

The purpose of this interpolator is to make the interpolation curve smoother and improving the image edges.

The cubic B-spline function is defined in Eq. (4):

$$f(x) = \sum_{k=-\infty}^{\infty} B_{k,n+1}(x) \cdot f(x_k). \tag{4}$$

The three-order B-spline function is as follows:

$$B_{i,3} = \begin{cases} \frac{(x-x_i)^2}{(x_{i+1}-x_i)(x_{i+2}-x)(x-x_{i+1})}. & ,x_i \leq x \leq x_{i+1} \\ \frac{(x-x_i)^2}{(x_{i+1}-x_i)(x_{i+2}-x_i)} - \frac{(x_{i+2}-x)(x-x_{i+1})}{(x_{i+2}-x_{i+1})(x_{i+3}-x_{i+1})}. & ,x_{i+1} \leq x \leq x_{i+2} \\ \frac{(x-x_{i+3})^2}{(x_{i+3}-x_{i+1})(x_{i+3}-x_{i+2})}. & ,x_{i+2} \leq x \leq x_{i+3} \end{cases} \tag{5}$$

The B-spline interpolator has a greater mathematical complexity, because the base functions do not support an explicit expression and change when adjusting the nodes vector.

3 Methodology

For this work and because they are analyzing depth images it is decided to choose two objects that have the following properties.

1. Rigid object of known form, non-deformable.
2. Soft and amorphous object.

This is considered as the most special cases that can be found when acquiring an image through a ToF camera.

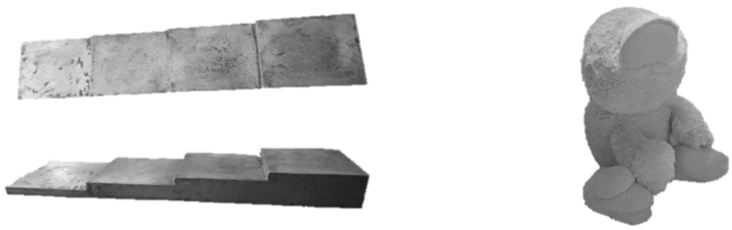


Fig. 1. Objects acquired with the depth sensor

Depth images were acquired with a ToF camera, to reduce the noise generated by the sensor, 250 depth images were acquired and we averaged to estimate the expected value of each pixel.

Were implemented and applied the three interpolators to the acquired images (Linear, Lagrange, and B-spline), increasing the acquisitions two, four, eight and sixteen times, to analyze the effect of each interpolator and each increase over the information provided in the original image.

To describe the original image, we calculated the central moments and compared with the central moments of the interpolated images. The formula of the central moment of order k is described in Eq. (6):

$$\mu_k = E[(X - E[X])^k]. \tag{6}$$

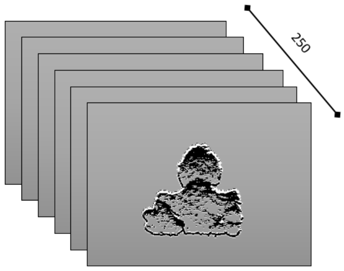


Fig. 2. Frames acquired by object

Where E is the expectation operator and k is the order of the statistical moment that is being calculated.

4 Results

In this section, we present the results obtained by applying the three different interpolators on the acquired original image, both qualitative and quantitative:

In the Figure 3 and 4 the interpolation applied to the original images of the plush doll and the calibration pattern respectively is shown, in both only a fragment of the image is shown as an example so that the effect of the interpolation can be better observed.

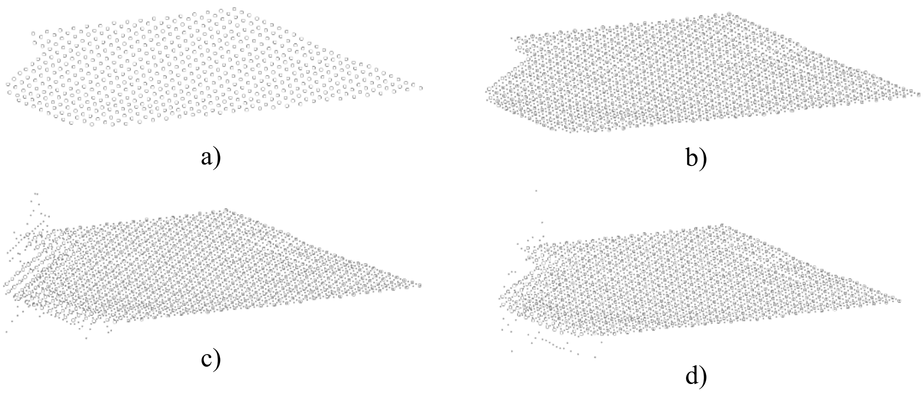


Fig. 3. (a) Original image (plush doll), (b) Image with Linear Interpolator (plush doll), (c) Image with Lagrange Interpolator (plush doll), (d) Image with B-Spline Interpolator (plush doll)

In tables 1-6 we present the descriptors of the first four central moments to compare the interpolated images with the input image. Row OI represents the information obtained from the original image, and the subsequent rows represent the central moments of the interpolated images by increasing the frequency 2, 4, 8 and 16 times the original with each interpolator.

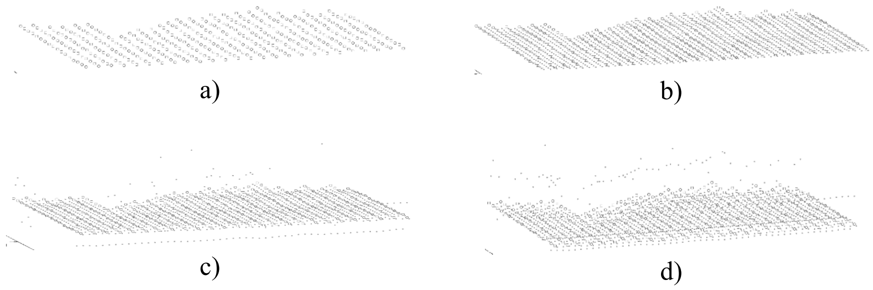


Fig. 4. (a) Original image (pattern depth), (b) Image with Linear Interpolator (pattern depth), (c) Image with Lagrange Interpolator (pattern depth), (d) Image with B-Spline Interpolator (pattern depth)

Figures 5,6 and 7 depict visually how the moments of the scaled images vary more frequently with respect to the original moments of the depth image(plush doll).

Table 1. Linear interpolation (plush doll)

	1^{st} M	2^{nd} M	3^{rd} M	4^{th} M
OI	0.055920057	4567.730957	106484.9375	*76746880*
2x	0.058058724	4518.415039	111223.5469	75409888
4x	0.291386068	4520.250488	111087.3047	75139080
8x	2.15897727	4531.534668	146713.9219	76388360
16x	8.629109383	4570.442383	1114.875366	72706552

Table 2. B-Spline interpolation (plush doll)

	1^{st} M	2^{nd} M	3^{rd} M	4^{th} M
OI	0.055920057	4567.730957	106484.9375	76746880
2x	0.01438925	5114.751465	1607.946045	128895192
4x	0.210121885	5103.547852	28430.24414	129927656
8x	2.253632307	5101.760254	7790.346191	129600160
16x	7.980696201	5140.467285	149802.7813	132150752

Table 3. Lagrange interpolation (plush doll)

	1^{st} M	2^{nd} M	3^{rd} M	4^{th} M
OI	0.0559200569987	4567.73095703125	106484.937500000	76746880
2x	0.0138180907815	5374.58984375000	242783.953125000	261666128
4x	0.2379906475543	5420.54345703125	230595.296875000	246231024
8x	2.1160101890564	5416.71679687500	186943.125000000	241594672
16x	8.2407598495483	5457.23486328125	352931.562500000	250968080

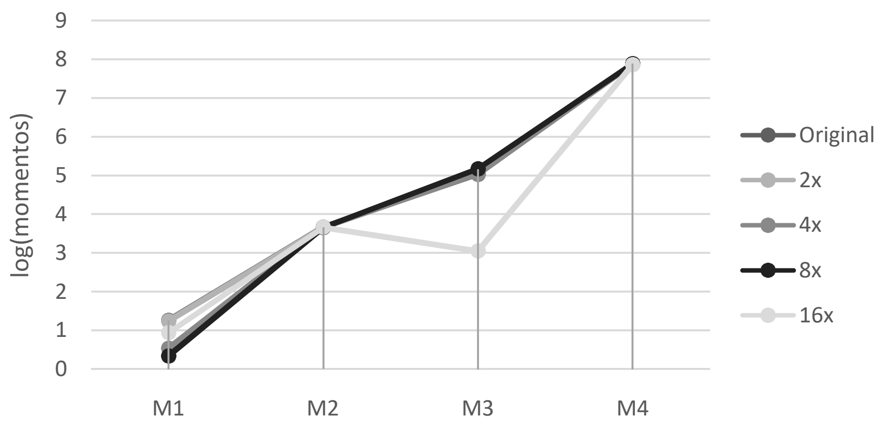


Fig. 5. Lineal interpolator

In Figure 5 it is observed that the linear interpolator for this type of images, offers good results, however, when we scale 16x the original data a considerable variation is observed for the moment 3.

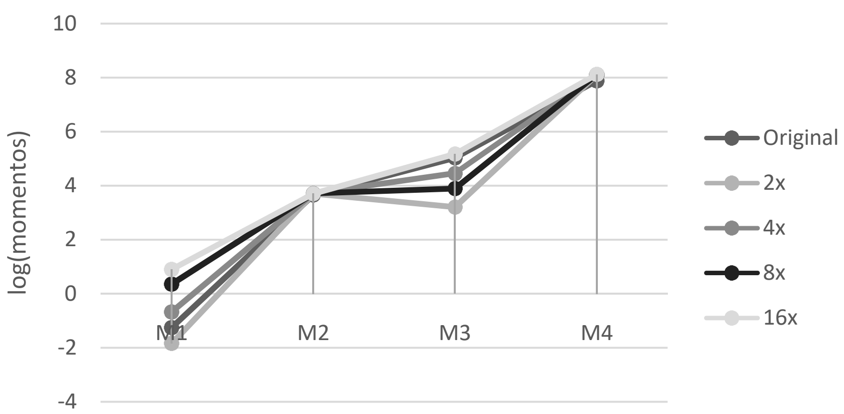


Fig. 6. B-Spline interpolator

Both the B-Spline interpolator and the Lagrange interpolator, from scaling 2x, show a noticeable variation with respect to the original data; however, the B-Spline interpolator as seen in Figure 6 varies more than the Lagrange interpolator Figure 7 for moment 3.

Figures 5, 6 and 7 depict visually how the moments of the scaled images vary more frequently with respect to the original moments of the depth image(pattern of depth).

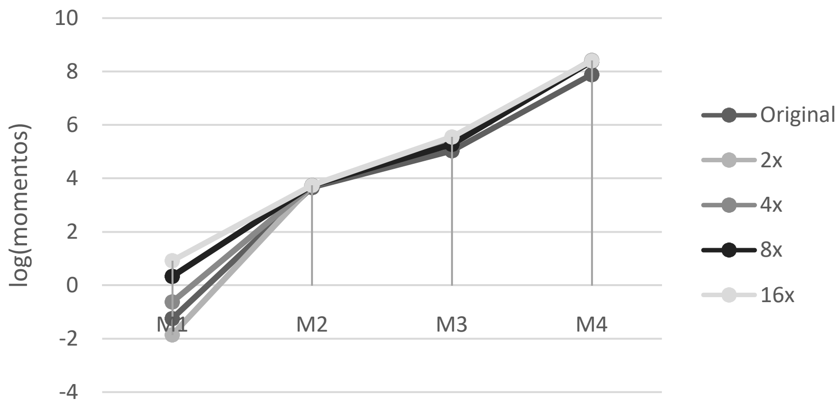


Fig. 7. c) Lagrange interpolator

Table 4. Liner interpolation (pattern of depth)

	1st M	2nd M	3rd M	4th M
OI	0.0001281092700	60.22936633	569.4663696	11182.77637
2x	0.0001294422447	62.19714737	555.3723755	10825.80664
4x	0.000847409	63.29848484	524.324462890	10410.88477
8x	0.00371421	65.824699401	763.623840332	13556.16309
16x	0.047592497	84.282928466	1455.68054199	27358.07422

Table 5. B-Spline interpolation (pattern of depth)

	1st M	2nd M	3rd M	4th M
OI	0.0001281092700082	60.22936630249	569.46636962890	11182.7763
2x	0.0092535801231861	3264.8283691406	612950.50000000	174173216
4x	0.0305372662842274	3582.5812988281	626727.62500000	171449104
8x	0.204221203923225	3606.1777343750	628083.93750000	172152064
16x	3.49978852272034	3622.1728515625	588958.06250000	163265120

Table 6. Lagrange interpolation (pattern of depth)

	1st M	2nd M	3rd M	4th M
OI	0.0000000	0.0060229	0.0569466	1.1182776
2x	0.0000000	0.0000562	0.0175613	6.9509043
4x	0.0000000	0.0000605	0.0165664	6.1430662
8x	0.0000000	0.0000608	0.0164137	6.1218336
16x	0.0000000	0.0000609	0.0158865	5.9289088

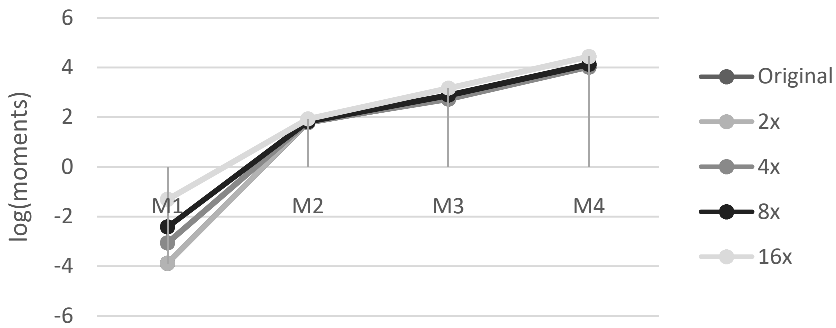


Fig. 8. Lineal interpolator

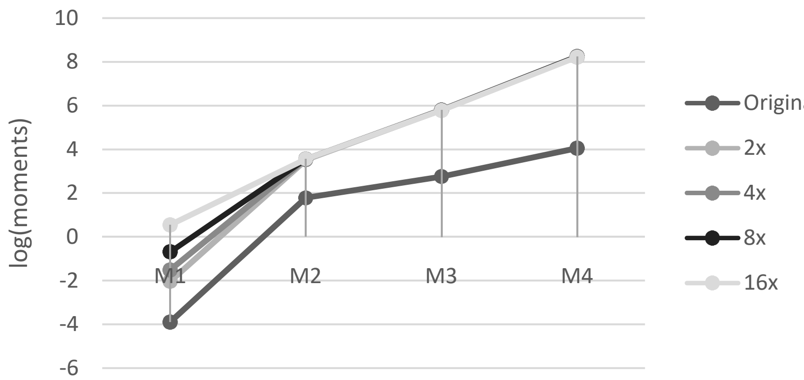


Fig. 9. B-Spline interpolator

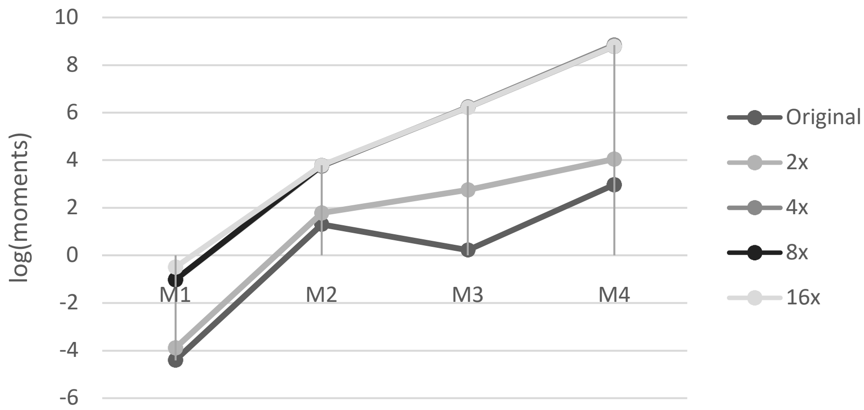
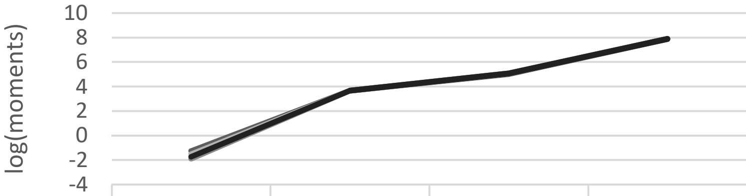
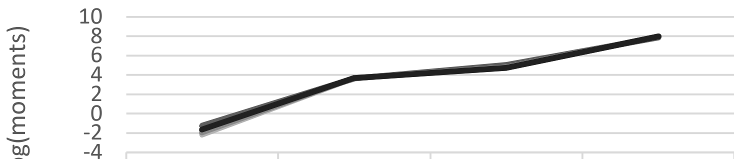


Fig. 10. Lagrange interpolator



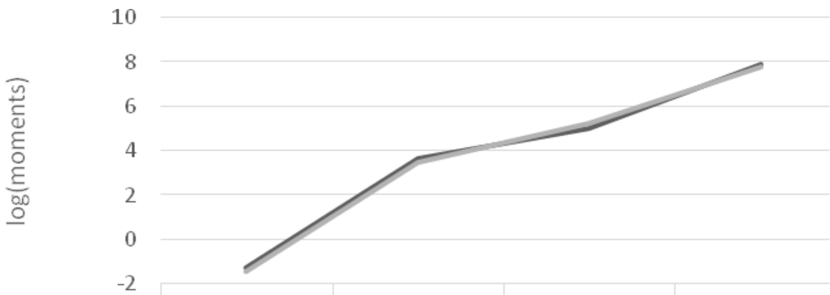
	M1	M2	M3	M4
0.1	-1.252432395	3.659700516	5.02728818	7.885060729
0.2	-1.524917387	3.654256703	5.068531744	7.873274275
0.3	-1.889672726	3.652817907	5.058930548	7.87180081
0.4	-1.723817828	3.653564623	5.062984278	7.872366129

Fig. 11. Linear Interpolation



	M1	M2	M3	M4
0.65	-1.252432395	3.659700516	5.02728818	7.885060729
0.75	-2.189894425	3.683789617	4.75175115	7.981087726
0.85	-1.955890124	3.682649578	4.740365003	7.980698487
0.95	-1.671134563	3.683023852	4.740945296	7.979722859

Fig. 12. B-Spline Interpolation



	M1	M2	M3	M4
Original	-1.252432395	3.659700516	5.02728818	7.885060729
0.2	-1.440779	3.469215642	5.220424223	7.795525155

Fig. 13. Lagrange Interpolation

5 Conclusions

In this project, we performed a comparison between the variation of the central moments initials of a depth image against the central moments obtained from a processed image using three different interpolation methods.

Quantitatively it can be observed that there is variation in the central moments of the original image given the selected interpolator as seen in Tables 1-6; however, it is also observed that it depends on the shape of the object being analyzed.

As can be seen in Fig. 5.6 and 7 it can be seen that the linear interpolator is the one that best fits the original central moments of the image, however, increasing it by 16x for the moment 3 is considerably different from the original.

As observed both the Lagrange interpolator and the B-Spline interpolator are the one that presents the greatest variation with respect to the original information as seen in Figure 6, 7, 9 and 10.

Because of this, the image is scaled with a smaller frequency to be able to visualize where it begins to move away from the original moments Figure 11, 12, 13, so that when processing images of depth from ToF sensors, this is taken into consideration.

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